

# **Experimental Study on the Small Strain Shear Modulus of Unsaturated Soils**

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the degree of

**Doctor of Philosophy**

under the supervision of A/Prof Behzad Fatahi and A/Prof  
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## **CERTIFICATE OF ORIGINAL AUTHORSHIP**

I, Thang Pham Ngoc, declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology at the University of Technology Sydney. This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis. This document has not been submitted for qualifications at any other academic institution.

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## DEDICATION

This thesis is deeply dedicated to the following people:

*To my respective parents, Nguyen Thi Thanh & Pham Ngoc Hien*

For their sacrifices, endless love, hard work, and inspiration

*To my wonderful wife, Ngoc Nguyen Bich*

For her constant love, support and caring

*To my lovely daughters, Giang Pham Chau & Ha Pham Ngoc*

For bringing joy and hope

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## ABSTRACT

The small strain shear modulus ( $G_{max}$ ) is a key parameter in analysing and predicting the dynamic response of soils. Numerous experimental studies have been conducted to investigate  $G_{max}$  of unsaturated soils during drying and wetting processes and loading and unloading stages of net stress, however, limitations still exist, requiring more research studies in this field. Specifically, the impact of hydraulic hysteresis, an important characteristic of unsaturated soil experiencing a number of drying-wetting and loading-unloading cycles of net stress, has not been captured successfully. Another issue in existing research studies is that the variation of  $G_{max}$  during an air-drying process was measured without considering the effect of matric suction equalisation, resulting in misrepresentation of the response of the material. In addition, the effects of drying-wetting and loading-unloading cycles on  $G_{max}$  of cemented soils have received little attention, and thus need to be investigated.

In this study, experimental studies were conducted to address the above-mentioned issues. To investigate the impact of hydraulic hysteresis on  $G_{max}$  of an unsaturated reconstituted soil, the specimen was subjected to a multi-stage test during several drying-wetting cycles and a loading-unloading cycle of net stress using a modified unsaturated Rowe cell apparatus. The results revealed four key factors that directly influence the magnitude of  $G_{max}$ : the void ratio, the net stress, matric suction and degree of saturation. While variations of the void ratio, net stress, and matric suction cause persistent responses of  $G_{max}$  (i.e. if all other factors remain unchanged,  $G_{max}$  would then be reversely proportional to the void ratio and directly proportional to the net stress and matric suction), variations in the degree of saturation result in different responses. A decrease in the degree of saturation may induce a reduction or growth of  $G_{max}$  since on the one hand, it reduces the effect of matric suction, while on the other hand, it increases the total effect of van der Waals attractions and electric double layer repulsions.

An analysis of the results showed that hydraulic hysteresis occurred in all the stress loops, and it directly influenced the response of  $G_{max}$ . The effect of hydraulic hysteresis can only be captured if the van der Waals attractions and electric double layer repulsions are considered. A model to estimate  $G_{max}$  while incorporating the van der Waals attractions and electric double layer repulsions has been proposed and it provided a good agreement with the experimental measurements. For practical issues, this model allows the determination of  $G_{max}$  of unsaturated soils based on the stress state, void ratio, and degree of saturation regardless of the stress and drying-wetting history, thus improving the accuracy of capturing response of  $G_{max}$  in complex loading-unloading and drying-wetting cycles.

To investigate the impact of matric suction equalisation on the measurement of  $G_{max}$  during an air-drying process, a weight-controlled bender element test was developed allowing the evolution of  $G_{max}$  at each test stage until matric suction equalisation is reached. Test results indicated that excluding matric suction equalisation causes underestimation of  $G_{max}$  measurement, especially in the middle range of the degree of saturation. This underestimation could be due to the non-uniform distribution of the water content and the corresponding matric suction across the cross section of the soil sample. The impact of matric suction equalisation on the measurement of  $G_{max}$  was rather small in the early stages of the air-drying process but accelerated when degree of saturation approached the threshold corresponding to the shrinkage limit. It was believed that water discontinuity, developed at water content smaller than the shrinkage limit, prevented the hydraulic flow from diminishing the non-uniform distribution of the water content, and thus, caused the underestimation of  $G_{max}$ . It was also found that effect of the unsaturated coefficient of permeability on the time required for matric suction equalisation is significant only at high degrees of saturation when the water phase is still continuous, while at lower degrees of saturation, when the water phase loses its continuity, effect of the unsaturated coefficient of permeability is gradually overcome by the effect of

water evaporation. Two empirical equations were proposed to determine the time required for matric suction equalisation in experiments studying behaviours of unsaturated soil after air drying and to predict  $G_{max}$  of compacted soil layers near the ground surface which can experience significant evaporation when exposed to the open environment.

To investigate effects of drying-wetting and loading-unloading cycles on  $G_{max}$  of cemented soils, a cemented sample cured under a constant stress was subjected to a multi-stage test during several drying-wetting and loading-unloading cycles using the modified unsaturated Rowe cell apparatus. The test results revealed that drying and wetting caused degradation of cementation of lightly cemented soil. The cementation degradation reduced the contribution of cementation and increased the contribution of the stress state to  $G_{max}$ . It was observed that, under constant matric suction, drying and wetting also occurred during the loading and unloading stages, respectively, due to the changes of the pore water pressure with loading-induced contraction and unloading-induced swelling. Consequently, degradation of cementation increased with increasing number of drying-wetting cycles as well as loading-unloading cycles. With an increase in the number of drying-wetting and loading-unloading cycles,  $G_{max}$  at the high stresses would intensify due to an increase in the contribution of the stress state, while  $G_{max}$  at the low stresses would decrease due to the reduction in the contribution of the cementation. In general, small strain shear modulus of lightly cemented soil was influenced by void ratio, cementation, degree of cementation degradation, and stress level. The effects of these influencing factors varied during drying-wetting cycles as well as loading-unloading cycles of net stress, thus, stress-strain history plays an important role in predicting the  $G_{max}$  of lightly cemented soils on site.

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## NOTATIONS

### Latin notations

$A$	Material constant
$b_1$	Fitting parameter
$b_2$	Fitting parameter
$c$	Fitting parameter
$c_k$	Permeability change index
$C(\psi)$	Correction function
$e$	Void ratio
$e_{min}$	The minimum void ratio at the end of the zero shrinkage stage
$e_{ref}$	Void ratio at a reference point
$f(e)$	Void ratio function
$G_{max}$	Small strain shear modulus
$G_s$	Specific gravity
$L$	Propagation distance
$m$	Fitting parameter
$m_1$	Fitting parameter
$m_2$	Fitting parameter
$n$	Soil porosity
$N$	Net stress
$k$	Fitting parameter
$k_{ref}$	Saturated hydraulic conductivity at a reference point
$k_s$	Saturated hydraulic conductivity
$k_w$	Unsaturated hydraulic conductivity

$p_a$	Atmospheric pressure
$p_r$	Reference pressure
$p'$	Mean effective stress
$S_e$	Effective degree of saturation
$S_r$	Degree of saturation
$S_{rs}$	Degree of saturation corresponding to the shrinkage limit
$S_{r0}$	Degree of saturation at saturated state
$S_{r,res}$	Residual degree of saturation
$t$	Propagation time
$t_{se}$	Time required for matric suction equalisation
$u_a$	Pore air pressure
$u_w$	Pore water pressure
$u_a - u_w$	Matric suction
$V_s$	Shear wave velocity
$W$	Gravimetric water content
$W_L$	Liquid limit

### **Greek notations**

$\alpha$	Fitting parameter
$\beta$	Fitting parameter
$\gamma$	Fitting parameter
$\rho_{eff}$	Effective dynamic mass density
$\sigma_{vn}$	Vertical net stress
$\sigma'_v$	Vertical effective stress
$\chi$	Effective stress parameter

$\chi_m$	Effective stress parameter for matric suction
$\chi_s$	Effective stress parameter for solute suction
$\psi$	Matric suction
$\psi_r$	Residual matric suction

## Acronyms

<i>AEV</i>	Air entry value
<i>HAEPD</i>	High air entry value porous disk
<i>LVDT</i>	Linear variable differential transformer
OCR	Over consolidation ratio
<i>SWCC</i>	Soil water characteristic curve